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Light Dirac Neutrinos In An $SU(2)_L \times U(1)_Y$ Model

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Light Dirac neutrino of mass of mass of the order of few eV is obtained in an $SU(2)_L \times U(1)_Y$ model with an extended Higgs sector and right-handed neutrinos. Small neutrino mass is generated at the tree level through small effective coupling of the Dirac neutrino mass term due to soft discrete symmetry breaking. In order to remove the exact degeneracy in mass between the second and the third generation of neutrinos, one loop corrected mass terms are incorporated. The model can accommodate bi-maximal mixing scenario of neutrino which has been favoured by recent Solar and atmospheric neutrino experiments.

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The long standing conjecture of neutrino oscillation as well as non-zero neutrino mass has recently been favoured by the result of Atmospheric neutrino experiment observed by the Super-Kamiokande Collaboration [1]. The results of SOUDAN [2] and CHOOZ [3] experiments are also consistent with the Super-Kamiokande experimental results. The reported result of Super-Kamiokande experiment leads to the oscillation of $\nu_\mu \rightarrow \nu_\tau$ (or ν_s) with the mass-squared diference $\Delta m^2_{\mu\tau} \sim 5 \times 10^{-4} - 6 \times 10^{-3}$ eV^2 as well as the mixing between the neutrino species is maximal. Furthermore, the solar neutrino problem could be resolved by the $\nu_e \rightarrow \nu_\mu$ oscillation with $\Delta {m^2}_{e\mu} \sim (0.3 - 0.7) \times 10^{-5}$ eV^2 and $Sin^2 2\theta_{e\mu} \sim 3.5 \times 10^{-3}$ if the small angle MSW solution is considered. The large angle MSW solution predicts the value of $\Delta {m^2}_{e\mu} \sim 10^{-5} - 10^{-4}$ eV^2 and $Sin^2 2\theta_{e\mu} \sim$ 0.8 - 1 [4].

In order to reconcile simultaneously the solar and the atmosheric neutrino problem as well as the candidature of neutrino as a hot dark matter component in a mixed dark matter scenario it has been pointed out [5] that the three electroweak neutrinos are almost degenerate in mass of the order of few eV. Explicit realization of almost degenerate neutrino mass has been demonstrated by several authors [5,6]. Furthermore, the present solar and atmospheric neutrino experimental data suggests the bi-maximal mixing pattern [7] that is maximal mixing between ν_e -ν_μ and ν_μ - ν_τ.

In the present work, we demonstrate that an $SU(2)_L \times U(1)_Y$ model with discrete $S_3 \times Z_3 \times Z_4$ symmetry, right-handed neutrinos and appropriate Higgs fields can give rise to almost degenerate Dirac neutrino of mass of the order of few eV as well as 'bi-maximal' mixing pattern in the neutrino- charged lepton charged current interactions. The scenario of degenerate Dirac neutrino

has recently been investigated [8] in view of Super-Kamiokande experimental result. In general, the Dirac neutrinos are much heavier than the Majorana neutrinos unless by some mechanism the Yukawa couplings associated with the Dirac neutrino mass terms are made smaller. In the present work, Dirac neutrino of mass of the order of few eV is generated through the small effective coupling of the Dirac neutrino mass term. The methodology has been proposed in Ref.[9] in order to generate small Majorana neutrino mass in the context of an $SU(2)_L \times U(1)_Y$ model. The present model gives rise to light Dirac neutrino mass at the tree level as well as at the one loop level which can accommodate the solar and atmospheric neutrino experimental results. We have discarded the LSND observed neutrino experimental result [10] due to its mismatch with the KARMEN [11] neutrino experimental result.
We concentrate on the lepton and Higgs fields of the model. The lepton content in the present model is as follows

$$l_{iL} \ \ (2,\ -1,\ 1), \quad \nu_{iR} \ \ (1,\ 0,\ 1) \tag{1}$$

where $i = 1, 2, 3$ is the generation index. The first two digits in the parenthesis respectively represent $SU(2)_L$ and $U(1)_Y$ quantum numbers and the last digit represents lepton number $\mathrm{L}(= \mathrm{L}_e + \mathrm{L}_\mu + \mathrm{L}_\tau)$ which is conserved in the present model. The following Higgs fields are considered with the vacuum expectation values (VEV's) as indicated

$$\phi_i \ (2,\ 1,\ 0) \ = \ \begin{pmatrix} 0 \\ v_i \end{pmatrix}, \quad \eta \ \ (1,\ 0,\ 0) \ = \ k \tag{2}$$

where $i = 1,\ldots 4$ are the number of doublet Higgs fields, and η is a real scalar singlet. The singlet Higgs fields do not couple with the leptons at the tree level due to the discrete symmetry incorporated in the model.

The lepton and Higgs fields transform under discrete $S_3 \times Z_3 \times Z_4$ symmetry as follows:

i) <u>S_3 Symmetry</u>

$$
\begin{aligned}
(l_{2L}, l_{3L}) \rightarrow 2, \;\; l_{1L} \rightarrow 1, \;\; (\nu_{\tau R}, \nu_{\mu R}) \rightarrow 2 \\
\nu_{e_R} \rightarrow 1, (e_R \, , \, \mu_R) \rightarrow 2, \tau_R \rightarrow 1, \\
\phi_1 \rightarrow 1, \phi_2 \rightarrow 1, (\phi_3, \phi_4) \rightarrow 2, \eta \rightarrow 1
\end{aligned} \qquad (3)
$$

ii) <u>$Z_3 \times Z_4$ Symmetry</u>

$$
\begin{aligned}
(l_{2L}, l_{3L}) \rightarrow \omega^\star (l_{2L}, l_{3L}), l_{1L} \rightarrow l_{1L}, (\nu_{\tau R}, \nu_{\mu R}) \rightarrow \omega^\star (\nu_{\tau R}, \nu_{\mu R}), \\
\nu_{eR} \rightarrow \nu_{eR}, (e_R \, , \, \mu_R) \rightarrow -i\omega^\star (e_R \, , \, \mu_R), \tau_R \rightarrow -i\omega^\star \tau_R, \\
\phi_1 \rightarrow \phi_1, \phi_2 \rightarrow i\phi_2, (\phi_3, \phi_4) \rightarrow i\omega(\phi_3, \phi_4), \\
\eta \rightarrow -\eta
\end{aligned} \qquad (4)
$$

where $\omega = exp(2\pi i/3)$

The discrete symmetry gives rise to some vanishing elements in the Dirac neutrino mass matrix and the charged lepton mass matrix at the tree level. The purpose of incorporation of S_3 permutation symmetry is to generate the equality between the Yukawa couplings associated with the neutrinos in order to get degenerate neutrino mass. The exact degeneracy is lifted due to the incorporation of one loop corrected neutrino mass. The Higgs field ϕ_1 couples only with the neutrinos and is prohibited from coupling with the charged leptons. We will estimate the value of v_1 by utilizing the minimization condition of the Higgs potential. The ϕ_2 Higgs field couples only with

the charged leptons as well as gives rise to small neutrino mass through its coupling with ϕ_1 and η Higgs fields through soft discrete symmetry breaking term contained in the Higgs potential of the present model. The purpose of incorporation of ϕ_3 and ϕ_4 Higgs fields is to achieve non-degenerate charged lepton mass matrix. Apart from the electroweak symmetry breaking scale, the present model contains another intermediate symmetry breaking scale at which the non-zero VEV of η Higgs fields is developed. The smallness of the neutrino mass is controlled by the VEV of the singlet Higgs field and the coefficient of the soft discrete symmetry violating term relating ϕ_1, ϕ_2, η Higgs fields contained in the Higgs potential. The soft discrete symmetry breaking term is necessary to remove any extra U(1) global symmetry in the model. We also discard any hard discrete symmetry breaking term for our analysis.

The most general renormalizable, Higgs potential in the present model can be expressed as

$$V = V(\phi_i) + V(\eta) + V(\phi_i, \eta_j). \tag{5}$$

where $V(\eta)$ is not relevant for the present analysis and the rest of the terms are explicitly given by

$$\begin{gathered}
V(\phi_i) = m_1^2(\phi_1^\dagger \phi_1) - m_2^2(\phi_2^\dagger \phi_2) + \sum_{i=1}^{2} \mu_i (\phi_i^\dagger \phi_i)^2 - m_3^2(\phi_3^\dagger \phi_3 + \phi_4^\dagger \phi_4) + \\
\mu_3(\phi_3^\dagger \phi_3 + \phi_4^\dagger \phi_4)^2 + \lambda_1(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) \\
+\lambda_2(\phi_1^\dagger \phi_1)(\phi_3^\dagger \phi_3 + \phi_4^\dagger \phi_4) + \lambda_3(\phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3 + \phi_4^\dagger \phi_4) \\
+2\lambda_4(\phi_3^\dagger \phi_4 \phi_4^\dagger \phi_3)
\end{gathered} \tag{6}$$

and

$$V(\phi_i, \eta) = \sum_{i=1,..4} \lambda_{ii}(\phi_i^\dagger \phi_i)(\eta^2)\, (\text{with}\, \lambda_{33} = \lambda_{44}) + \lambda'(\phi_1^\dagger \phi_2 \eta + \phi_2^\dagger \phi_1 \eta) \tag{7}$$

It is to be noted that, in order to get neutrino mass $\sim$ eV, we consider the VEV of ϕ_1 is zero at the tree level. This has been achieved by choosing positive mass term ($m_1^2 > 0$) of the ϕ_1 Higgs field in Eq.(6). The non-zero VEV of ϕ_1, arising due to the presence of the soft discrete symmetry breaking term λ', is estimated as follows. Substituting the VEV's of the Higgs fields in Eqs.(6) and (7) and minimizing the entire Higgs potential with respect to v_1, we get

$$v_1 = -\frac{B}{A} \tag{8}$$

with

$$B = \lambda' v_2 k \tag{9}$$

$$A = m_1^2 + \lambda_1 v_2^2 + \lambda_2 (v_3^2 + v_4^2) + \lambda_{11} k^2 \tag{10}$$

where we have neglected μ_1 term for simplicity. On simplification of Eq.(8), we obtain

$$v_1 = -\frac{\lambda' v_2 k}{m_1^2} \tag{11}$$

assuming m_1^2 to be much larger than all other terms in Eq.(10). However, the above assumptions do not affect the essential results derived in our present analysis.

The most general discrete symmetry invariant lepton-Higgs Yukawa interaction, in our present model is as follows

$$L_Y \;=\; [f_1(\bar{l_{2L}} \nu_{\tau R} + \bar{l_{3L}} \nu_{\mu R}) + f_2 \bar{l_{1L}} \nu_{eR}] \tilde{\phi_1} +$$

$$
\begin{aligned}
&+ \quad g_1(\bar{l_{2L}} e_R + \bar{l_{3L}} \mu_R)\phi_2 + g_2(\bar{l_{2L}} \phi_3 + \bar{l_{3L}} \phi_4)\tau_R \\
&+ \quad g_3 \bar{l_{1L}} (e_R \phi_3 + \mu_R \phi_4) + H.c. \qquad (12)
\end{aligned}
$$

Substituting the VEV's of the Higgs fields in Eq.(12) the tree level Dirac neutrino mass matrix M_ν^0 comes out as

$$M_\nu^0 = \begin{pmatrix} \xi a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{pmatrix} \qquad (13)$$

where

$$a = f_1 v_1, \xi = \frac{f_2}{f_1} \qquad (14)$$

In order to remove the exact degeneracy between the mass of the second generation and the third generation , we have incorporated one-loop mass terms arising due to the charged Higgs exchange. The one-loop corrected neutrino mass matrix is given by

$$M_\nu' = \begin{pmatrix} x_1 & 0 & x_2 \\ x_3 & x_4 & 0 \\ 0 & x_5 & x_6 \end{pmatrix} \qquad (15)$$

where

$$x_1 = g_3 f_2 \lambda_2 v_3 v_1 m_e F(M_3^2, M_1^2) \qquad (16)$$

$$x_2 = g_3 f_1 \lambda_2 v_4 v_1 m_\mu F(M_4^2, M_1^2) \qquad (17)$$

$$x_3 = g_1 f_2 (\lambda' k + \lambda_1 v_1 v_2) m_e F(M_2^2, M_1^2) \qquad (18)$$

$$x_4 = g_2 f_1 \lambda_2 v_3 v_1 m_\tau F(M_3^2, M_1^2) \qquad (19)$$

$$x_5 = g_2 f_1 \lambda_2 v_1 v_4 m_\tau F(M_4^2, M_1^2) \qquad (20)$$

$$x_6 = g_1 f_1 (\lambda' k + \lambda_1 v_1 v_2) m_\mu F(M_2^2, M_1^1) \qquad (21)$$

and

$$F(M_i^2, M_j^2) = \frac{1}{16\pi^2(M_i^2 - M_j^2)} \ln\frac{\mathrm{M_i^2}}{\mathrm{M_j^2}} \tag{22}$$

and $M_i's$ are the masses of the charged Higgs fields. Combining Eq.(13) and (15), we get the total mass matrix of the neutrino as

$$M_\nu = M_\nu^0 + M_\nu' \tag{23}$$

The charged lepton mass matrix comes out from Eq.(12) as

$$M_l = \begin{pmatrix} g_4 v_3 & g_4 v_4 & 0 \\ g_2 v_2 & 0 & g_3 v_3 \\ 0 & g_2 v_2 & g_3 v_4 \end{pmatrix} \tag{24}$$

In this situtation, it is not possible to diagonalise simultaneously the neutrino mass matrix and the charged lepton mass matrix and the mismatch between the diagonalisation matrices will give rise to the CKM -type mixing matrix in the leptonic sector. Before going to diagonalise the neutrino mass matrix M_ν , we assume the following to simplify the diagonalisation procedure without altering any essential results. First of all, we neglect all the terms in M_ν' proportional to m_e and m_μ since these are small compared to the terms proportional to m_τ. We consider the weak eigenstates and the mass eigenstates are related by the following relation

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} = U_l \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} \tag{25}$$

where

$$U_l = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \tag{26}$$

and $c_{ij} = Cos\theta_{ij}$, $s_{ij} = Sin\theta_{ij}$, i,j =1,2,3 are the generation indices. A similar relation for the neutrino sector is also considered

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_\nu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{27}$$

where

$$U_\nu = \begin{pmatrix} c'_{12}c'_{13} & s'_{12}c'_{13} & s'_{13} \\ -s'_{12}c'_{23} - c'_{12}s'_{23}s'_{13} & c'_{12}c'_{23} - s'_{12}s'_{23}s'_{13} & s'_{23}c'_{13} \\ s'_{12}s'_{23} - c'_{12}c'_{23}s'_{13} & -c'_{12}s'_{23} - s'_{12}c'_{23}s'_{13} & c'_{23}c'_{13} \end{pmatrix} \tag{28}$$

and $c'_{ij} = Cos\theta'_{ij}$, $s'_{ij} = Sin\theta'_{ij}$. In order to obtain 'bi-maximal' scenario , we set the mixing angles as

$$\theta_{13} = 0, \theta_{23} = \frac{\pi}{8}, \theta_{12} = \frac{\pi}{4} \tag{29}$$

for the charged lepton sector and

$$\theta'_{13} = 0, \theta'_{23} = \frac{\pi}{8}, \theta'_{12} = 0 \tag{30}$$

for the neutrino sector. The zero value's of θ_{13}, θ'_{13} and θ'_{12} is obvious from the structure of the mass matrices, and we set the non-zero values by adjusting model parameters. The above choice of mixing angles give rise to the following mixing matrix V in the neutrino-charged lepton charged current interaction which is given by

$$V = U_\nu^\dagger U_l = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{31}$$

as desired to obtain ' bi-maximal' mixing scenario to explain solar and atmospheric neutrino experimental data. On diagonalisation of $M_\nu M_\nu^\dagger$, we obtain the following eigenvalues

$$m_{\nu_1}^2 = \xi^2 a^2 \tag{32}$$

$$m^2_{\nu_2} = \frac{(q+s)+\sqrt{(q-s)^2+4r^2}}{2} \tag{33}$$

$$m^2_{\nu_3} = \frac{(q+s)-\sqrt{(q-s)^2+4r^2}}{2} \tag{34}$$

where $q = a^2 + x_4^2$, $s = (a + x_5)^2$, $r = x_4(a + x_5)$ and the mixing angle θ'_{23} is given by

$$tan2\theta'_{23} = \frac{2r}{s-q} \tag{35}$$

Neglecting higher orders of x_4 and x_5, and assuming $M_4 \sim M_3$ and $M_4, M_3 >> M_1$, the mass-squared differences $m^2_{\nu_2} - m^2_{\nu_3}$, $m^2_{\nu_2} - m^2_{\nu_1}$ and the mixing angle θ'_{23}, can be simplified as

$$m^2_{\nu_2} - m^2_{\nu_3} \sim 2ax_4 \tag{36}$$

$$m^2_{\nu_2} - m^2_{\nu_1} \sim a^2(1-\xi^2) \tag{37}$$

$$tan2\theta'_{23} \sim \frac{v_3}{v_4} \tag{38}$$

In order to accommodate the atmospheric neutrino experimental data, we set the mass-squared difference at a typical value as $m^2_{\nu_2} - m^2_{\nu_3} \sim 5 \times 10^{-4}$ and $tan2\theta'_{23} \sim 1$. For a numerical estimation , we consider the following choices of model parameters as $M_4 \sim M_3 = 400$GeV, $M_1 \sim M_2 = 200$GeV, $v_2 \sim v_3 \sim v_4 = 100$ GeV, $k = 1$ TeV and $m_1 = 1$ TeV. The above choice of model parameters gives rise to the mass-squared difference as required to solve the atmospheric neutrino problem with the constraint on the couplings as $f_1^2 \lambda'^2 g_2 \lambda_2 \sim 0.5 \times 10^{-3}$. The parameter ξ is estimated from the solar neutrino experimental data and for $m^2_{\nu_2} - m^2_{\nu_1} \sim 10^{-5}$ (for large angle MSW solution), the parameter ξ comes out slightly less than unity.

In summary, we have demonstrated that an $SU(2)_L \times U(1)_Y$ model with $S_3\times$ $Z_3\times$ Z_4 discrete symmetry, right-handed neutrinos and appropriate

Higgs fields give rise to light Dirac neutrino of mass of the order of few eV. Small neutrino mass is generated at the tree level due to the small effective coupling of the Dirac neutrino mass terms through the incorporation of soft discrete symmetry breaking term. In order to remove the exact degeneracy in mass between the second and the third generation of neutrinos, we have incorporated one loop corrected mass terms. The present model accommodates the solar and atmospheric neutrino experimental result through bi-maximal mixing angle scenario with a reasonable choice of model parameters.

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